Latent Heat of Fusion of Ice

Aim   To calculate the Latent Heat of Fusion of Ice by measuring the temperature changes as it melts in warm water.

Equipment   100 mL Measuring Cylinder   250 mL Beaker

                         Data Logger   Thermometer Probe
          Ice Cube

Method

1)   Measure 100.0 mL of warm water (approx 60 ^{o}C) with the 100 mL measuring cylinder.

2)   Put the 100.0 mL of water into a 250 mL beaker.

3)   Collect an ice cube from the freezer in the Technicians' Room. Record the temperature of the ice from the thermometer on the freezer door.

4)   Record the temperature of the water using the temperature probe and transfer the ice to it as quickly as possible after collecting it.

5)   Stir the water ice mixture with the temperature probe and record the temperature of the water as soon as all the ice has just melted.

6)   Measure the new volume of water using the 100 mL measuring cylinder.

7)   Repeat the whole experiment twice more.

Data:

Specific Heat Capacity of Liquid Water   4.18 J g^{-1} K^{-1}

Specific Heat Capacity of Ice   2.03 J g^{-1} K^{-1}

Latent Enthalpy of Fusion of Ice   6.01 kJ mol^{-1}

A Data Table Showing the Relationship between Time Elapsed
and the Temperature of the Reaction

Time Sec, ±0.05sec	Temperature Trial One, °C, ±0.2°C	Temperature Trial Two, °C, ±0.2°C	Temperature Trial Three, °C, ±0.2°C
0.00	60.28	62.18	69.52
10.00	55.59	51.47	62.91
20.00	48.81	44.27	55.49
30.00	41.69	41.59	54.42
40.00	37.71	41.20*	54.39*
50.00	35.63		
60.00	34.79		
70.00	34.16		
80.00	33.95		
	33.90*		
Decrease in Temperature (±0.2°C + ±0.2°C) ±0.4°C	=60.28-33.90 =26.38	=62.18-41.20 =20.98	=69.52-39 =15.13

Initial Volume, ml, ±0.5ml	100	100	100
Final Volume, ml, ±1ml[1]	124	118	110
Increase in Volume, ml, ±1.5ml	24	18	10
Ice Temperature °C, ±1°C	-19	-19	-19

Observations: the water was initially steaming and the beaker was warm to the touch. As soon as the ice was added, it began to melt. After a certain interval, there was no ice visible, and it was deemed to have melted. The water was no longer steaming.

Note that the intervals of time were chosen to minimise the amount of data recorded unnecessarily. Entries marked with a * show the temperature recorded when all of the ice had melted and which occurred before the next time interval. The data has been rounded to 4 significant figures. Whilst the data was supplied by the data logger was to 8 significant figures, such a level of accuracy is futile, considering the random error on the thermometer. 4 significant figures was chosen, based on the random error in the readings.

---

[1] Consult the evaluation for an explanation of this greater than normal error.

Calculations

Let us calculate the averages for our calculation. There are two ways of conducting this: to calculate the average of each property (temperature change and volume); relying on the point that the error will cancel between them or to conduct the calculations 3 times before creating a final average. The former is unsuitable. The ice was of different weights throughout and no attempt was made to keep this consistent. Equally no attempt was made to chose a consistent starting temperature. Whilst this should not affect the final result, it does mean that the error of the values would be artificial and misleading. Therefore, I shall calculate each trial and then average the result generated.

Trial One

The first step is to calculate the energy lost in this experiment. This can be done according to the following formula. M is the mass of water, s is the specific heat capacity of water and ΔT is the change in temperature.

Q_l = m. s. ΔT

Q_l = 100 x 4.18 x 26.38 $= \pm \left(\frac{1}{100} x100\right) + \pm \left(\frac{0.4}{26.38} x100\right)$

Q_l= 11026.84 J $= \pm 2.52 \dots \%$

The energy gained by the water must be calculated. Firstly, the energy gained to heat the ice from -19°C to 0°C. Before, however, an important assumption has to be made: that the density of water is 1 g ml^{-1}. This means that the mass of ice is the same as the increase in volume, converted into grams. 24ml increase here means that the ice weighed 24g ±3g.

Q_{g1} = m. s_{ice}. ΔT

Q_{g1} = 24 x 2.03 x 19 $= \pm \left(\frac{1.5}{24} x100\right) + \pm \left(\frac{0.4}{19} x100\right)$

Q_{g1} = 925.68J $= \pm 8.3 \dots$

From this, the 24 gram of ice is heated as water from 0°C to 33.90°C.

Q_{g2} = m. s_{water}. ΔT

Q_{g2} = 24 x 4.18 x 33.90 $= \pm \left(\frac{1.5}{24} x100\right) + \pm \left(\frac{0.4}{33.90} x100\right)$

Q_{g2} = 3400.848 J $= \pm 7.42 \dots$

Knowing these two values, the energy gain ($Q_{g\ total}$) in the experiment can be calculated:

$Q_{g\ total}$ = Q_{g1} + Q_{g2}

$Q_{g\ total}$ = 925.68 + 3400.848 $= \pm 8.3 \dots + \pm 7.42 \dots$

$Q_{g\ total}$ = 4326.528 J $= \pm 15.72 \dots$

Knowing these values, the following formula can be used to calculate the latent heat of fusion of ice. This formula requires us to remember the principle that energy cannot be destroyed.

Latent heat of fusion + Energy gain = Energy lost

Latent heat of fusion = Energy lost- energy gain

L_f = 11026.84 J - 4326.528 J $= \pm 2.52 \ldots + \pm 15.72 \ldots$

L_f = 6700.312 J $= \pm 18.42 \ldots$

This value is per the amount of moles of ice used.

moles(Ice)   = mass(Ice)/molar mass (Ice) $= \pm \left(\frac{1.5}{24} x100 \right)$

                24.00/18.02

moles(Ice)   =1.33... $= \pm 6.25\%$

L_f = 6700.312 J per 1.33 mol $= 6.25\ldots\% + \pm 18.42 \ldots$

L_f= 5030.81…. J per mole $= 24.67\%$

These calculations must now be repeated for trials two and three. For the sake of brevity, all explanatory lines of working have been removed.

Trial Two

Q_l = 100 x 4.18 x 20.98 $= \pm \left(\frac{1}{100} x100 \right) + \pm \left(\frac{0.4}{20.98} x100 \right)$

Q_l= 8736.2 $= \pm 1.90 \ldots$

18ml increase means that the ice weighed 18g ±3g

Q_{g1} = m. s_{ice}. ΔT

Q_{g1} = 18 x 2.03 x 19 $= \pm \left(\frac{1.5}{18} x100 \right) + \pm \left(\frac{0.4}{19} x100 \right)$

Q_{g1} = 694.26 $= \pm 10.43\ldots$

From this, the 18 gram of ice is heated as water from 0°C to 41.2°C.

Q_{g2} = m. s_{water}. ΔT

Q_{g2} = 18 x 4.18 x 41.20 $= \pm \left(\frac{1.5}{18} x100 \right) + \pm \left(\frac{0.4}{41.20} x100 \right)$

Q_{g2} = 3099.888 J $= \pm 9.30\ldots$

$Q_{g\ total}$ = Q_{g1} + Q_{g2}

$Q_{g\ total}$ = 694.26+ 3099.888 $= \pm 10.43\ldots + \pm 9.30\ldots$

$Q_{g\ total}$ = 4326.528 J $= \pm 19.73\ldots$

Lf = 8736.2 J - 4326.528 J $= \pm1.90 \dots + \pm19.73 \dots$

Lf = 4409.672 J $= \pm21.63 \dots$

This value is per the amount of moles of ice used.

moles(Ice)   =18.00/18.02 $= \pm\left(\frac{1.5}{18}x100\right)$

moles(Ice)   =0.998…   =8.33…%

Lf = 4409.672 per 0.998… mol $=\pm21.63 \dots\% + \pm8.33 \dots\%$

Lf=4418.51… J per mole   =29.96%

Trial Three

Q_l = 100 x 4.18 x 15.13 $= \pm\left(\frac{1}{100}x100\right) + \pm\left(\frac{0.4}{15.13}x100\right)$

Q_l= 6324.34 $= \pm2.64 \dots\%$

10ml increase means that the ice weighed 10g ±3g

Q_{g1} = m. s_{ice}. ΔT

Q_{g1} = 10 x 2.03 x 19 $= \pm\left(\frac{1.5}{10}x100\right) + \pm\left(\frac{0.4}{19}x100\right)$

Q_{g1} = 391.4 $=\pm17.10\dots\%$

From this, the 18 gram of ice is heated as water from 0°C to 54.39°C.

Q_{g2} = m. s_{water}. ΔT

Q_{g2} = 10 x 4.18 x 54.39 $= \pm\left(\frac{1.5}{10}x100\right) + \pm\left(\frac{0.4}{41.20}x100\right)$

Q_{g2} = 2273.502 J $=\pm15.97\dots\%$

$Q_{g\ total}$ = Q_{g1} + Q_{g2}

$Q_{g\ total}$ = 391.4 + 2273.502 $=\pm17.10\dots + \pm15.97\dots$

$Q_{g\ total}$ = 2664.902 J $=\pm33.07\dots\%$

Lf = 6324.34J - 2664.902 J $= \pm2.64 \dots + \pm33.07$

Lf = 3659.44 J $= \pm35.71 \dots\%$

moles(Ice)   =10.00/18.02 $= \pm\left(\frac{1.5}{10}x100\right)$

moles(Ice)   =0.554…   =15.00%

Lf = 3659.44 per 0.554… mol $=15\% +\pm35.71 \dots\%$

Lf=6594.31… J per mole   =50.71%

Average Latent Heat

$$= \frac{6594.31... + 4418.51... + 5030.81...}{3} \qquad\qquad = \frac{50.71\% - 24.67\%}{2}$$

=5281.21 J per mole   **=±13.02….%**
=5281.21 J per mole ±686 J
=5.28 kJ per mole ±1kJ (1sf)

Percentage Deviation

$$\frac{5281.21 - 6010}{6010} \times 100 = 12.12\%$$

Conclusion

 The aim of this experiment was to empirically determine the latent heat of fusion of ice and hence compare this to the accepted literature value. Accordingly, the hypothesis can be deemed to have been answered: we have empirically demonstrated that there is latent heat within ice, and calculated the latent heat of the ice. There was a significant percentage deviation from the literature value, however this can be ascribed to failings in the experiment. Over the three trials, the average latent het of the ice has been found to be 5.28kJ per mole. This compares to the literature figure of 6.01kJ per mole; a percentage deviation of 12.12%.

The percentage error is slightly smaller than the random error of this experiment. The random error was reasonably significant: a fact that can be explained by a poor choice of instruments as well as a method that involved significant internal random errors. The remaining error can be explained by systemic errors in this experiment. Whilst both are a factor here, the former, random error, is significantly more important.

Why did these results occur? The theory of latent heat is relatively simple. In a solid, the molecules in a substance have a relative position in space to each other. This explains the fact that a solid appears solid. In order to create this fixed shape, energy is required. When a solid melts, this shape collapses and the molecules no longer have a relative position to each other; thus releasing energy. This released energy is known as the enthalpy of fusion. In our experiment, as the ice melted, the energy that was involved in keeping the ice a solid was released when it melted. By using the calculations above, we were able to work out the extent of this heat release, and thus the amount of latent heat present.

Evaluation

The data that I have generated has limited statistical validity. The variation in results between trials was quite significant: the final error was ± 1kJ. This suggests problems in the method of my experiment, as well as random error. The average latent heat value was reasonably similar to the literature value: there was a reasonably low percentage deviation. However, what percentage deviation there was can be explained by reference to the methodological problems in this experiment.

One problem in this experiment was the possibility for heat loss. All enthalpy reactions assume that the reaction takes place in a closed system with no heat loss. In this case, in order for the calculations to be performed, it had to be assumed that the energy lost was equal to the energy gained plus the latent heat. In reality, this was not the case. The real equation would look something like this:

Latent heat of fusion + Energy gain = Energy lost + heat lost to surroundings.

The beaker that was used was completely uninsulated: no attempt was made to prevent heat loss through the glass to the surroundings. Had no heat have been wasted, cooling the surrounding air, the water would have become cooler than recorded. This explains why our values for latent heat of fusion were too low. Our value for energy lost was too low, as energy had been removed from the system in the form of heat loss to the surroundings. The value of energy lost should have been much higher, as, in theory, no heat should be lost to the surroundings. When rearranged, a higher value of energy lost would lead to a higher value of latent heat.

It is certainly easy to reduce heat loss. Most importantly, a glass beaker should not be used for this experiment: it has a very low insulating capacity. Instead, a vacuum flask should be used, with a much greater insulating capacity. This would have reduced the heat lost to the surrounds, increasing the value of energy lost and hence of latent heat. This may not be possible; therefore, the experiment should be conducted in a beaker of water of known temperature. The temperature change could be recorded, and hence an estimate gained of how much energy was lost to the surroundings, increasing the accuracy of the results considerably.

A key problem with this equipment was the use of a measuring cylinder. This was used to gauge the final and initial volume of the water, and hence calculate the weight of the ice. There are two problems with a measuring cylinder in this experiment.

Firstly, the measuring cylinder had a maximum capacity of 100ml. The final volume was always greater than 100ml. This meant that two measurements had to be taken: doubling the percentage error. Equally, a measuring cylinder is accurate to ±0.5ml. This is a reasonably high error, especially when better solutions are available. Both of these measures had the effect of increasing the random error in this experiment significantly. They also go some way to explaining the variation in latent heat.

Fortunately, the solution is simple. There is no need to record the volume of the water: it was only recorded to allow the weight of the ice to be recorded. It would be much simpler, as well as decreasing the possibility for error considerably, were the ice to be weighed prior to the experiment. This would significantly decrease the random error in the experiment, as well as increasing the precision of the answers.

The second source of random error was unfortunately inherent in this method. When two values are subtracted, the associated errors are added, according to the principle of error propagation. This has the unfortunate impact of increasing the error in a value, whilst decreasing the value of the figure. The result is to significantly increase the percentage error as a proportion of the result. For example:

10ml ±1ml – 7ml ±1ml = 3ml ±2ml

This is, unfortunately, very hard to remove from the experiment: we cannot calculate the enthalpy of fusion without a subtraction, and the measuring cylinder required doubling of errors in the same way. Whilst the

measuring cylinder aspect can be improved, it would be hard to improve the method to remove subtraction from it: it is a factor that must be remembered when considering the extent of the percentage error.

In each trial, we assumed that the ice was initially -19°C; the temperature of the freezer. However, this was only the temperature of the ice when it was in the freezer. As soon as it was removed, the ambient temperature of the room began the process of melting the ice. This means that the temperature of ice at the beginning of the reaction would be greater than recorded. This means that the enthalpy change of the ice (Q_{g1}) was greater than it should have been. Hence, the total energy gain would have been greater than expected and so too much was subtracted from the energy loss. This resulted in latent heat of fusion values that were too small.

Logistically, this a tricky problem to solve. The easiest solution would be one of control: the surface temperature of the ice could be recorded prior to the experiment and this noted. However, this is an imperfect solution: it would be hard to get a constant temperature, as the ice would continue to melt. The most practical solution would be to conduct the experiment next to the freezer. The ice could be very quickly transferred into the beaker and the experiment begun immediately. There would still be a small amount of melting, but it would be negligible compared to the current method.

An assumption made was that the ice used was pure frozen distilled water without any impurities. This is almost certainly not the case: the water is very unlikely to be completely pure. However, our calculations were based on the melting point of ice being 0°C. If the ice was impure, this would change the decrease the melting point to below 0°C (one reason why salt-water lakes rarely freeze). This means that the overall enthalpy gain figure was too low: too much water was multiplied by the smaller heat capacity of ice.

Another assumption was made that is similar: by assuming the specific heat capacity of water and ice, we assumed that the water was entirely pure. This was probably not the case, as previously mentioned. With impurities in the water, the specific heat constant should have been increased, increasing the enthalpy gain calculations.

The solution to this is simple. Prior to the experiment, distilled water should be used to create the ice that is to be used. Whilst distilled water will not remove all impurities, it will remove a very significant proportion of them, decreasing the percentage deviation.

A final comment is worthwhile on the conditions that the experiment was conducted in. The conditions that the experiment was undertaken in were not standard conditions: no effort was made to monitor temperature or pressure. However, the literature value is correct only at 298K and 1atm. We don't know the difference between ambient temperature of the experiment and standard temperature and pressure. However, if there was a significant difference in heat or pressure, this would create a lower value for latent heat, and hence explain the discrepancy from the percentage error.

An initial step can be taken to monitor the temperature of the laboratory using a thermometer. (Pressure is much trickier to monitor) The temperature being monitored, we can compare the difference between the standard conditions and the actual conditions. If this was statistically significant, it would help to explain the percentage deviation. If the differences were found to be so, a heat pupmp could be used to create the desirable temperature in the lab. This is, however, a crude device (it is unlikely that that 298K could be recreated) and is a less important measure than others in this experiment.